

SOME ENERGY CHARACTERISTICS OF THE PROCESS OF HEATING OF A POROUS LAYER BY AN INCOMPRESSIBLE LIQUID OR GAS FLOW

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The process of heating of a porous body that is initially at a low temperature by an incompressible liquid or gas flow is considered. The amount of heat energy that cannot be transferred to the porous layer because of the temperature difference between the liquid (or gas) and solid phases, respectively, is calculated. This energy may be considered "excess" energy. Its quantity is calculated analytically for the case of heating of a semi-infinite porous body.

One of the most important applications of porous media is their use in thermal energy accumulation systems. Calculation of heat- and mass-transfer processes in porous bodies is important for increasing the operational efficiency of such systems. It is therefore not surprising that the contemporary foreign literature gives close attention to the simulation of processes of heat and mass transfer in porous media. In [1-4] we find consideration and numerical investigation of a very general system of differential equations that describe forced convection of liquid or gas through a porous body in the absence of thermal equilibrium between the liquid (gaseous) and solid phases. Based on this model, some energy characteristics of the process of heat accumulation in a porous layer were investigated numerically in [5].

Below, an analytical study of the process of heating of a semi-infinite porous body by an incompressible liquid (gas) flow is performed. A two-phase model of the porous body is used that includes two energy equations: for the liquid and solid phases. The amount of heat that cannot be transferred to the porous matrix by the liquid (gaseous) phase because of the temperature difference between the phases is calculated. This heat cannot be accumulated by the solid phase and is therefore considered "excess" thermal energy. To the knowledge of the present author, this work is the first attempt to calculate the amount of excess energy in heating of a porous body.

It should be emphasized that previous analytical investigations of processes of heat and mass transfer in a porous body carried out on the basis of a model with two energy equations [6-10] employed productively the model of a porous medium suggested by T. Schumann [11], in which the thermal conductivity coefficients are neglected in the energy equations for both the liquid (gaseous) and solid phases. In the present work, the energy equations are taken in their complete forms, without ignoring any of their terms.

Mathematical Model. To formulate a model for investigation of the problem the following assumptions are used:

- the liquid (gaseous) phase is incompressible, and the flow rate in any cross-section of the porous body is constant;
- the thermophysical properties of the liquid (gaseous) and solid phases are invariant;
- heat transfer and liquid (gas) flow are one-dimensional.

Under these assumptions, the mathematical model of [3] is simplified to two energy equations for the liquid (gaseous) and solid phases, respectively:

$$\epsilon \langle \rho_f \rangle^f (c_p)_f \frac{\partial \langle T_f \rangle^f}{\partial t} + \langle \rho_f \rangle^f (c_p)_f \langle v_f \rangle \frac{\partial \langle T_f \rangle^f}{\partial x} = \lambda_{\text{feff}} \frac{\partial^2 \langle T_f \rangle^f}{\partial x^2} + h_{\text{sf}} a_{\text{sf}} [\langle T_s \rangle^s - \langle T_f \rangle^f], \quad (1)$$

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$$(1 - \varepsilon) \langle \rho_s \rangle^s (c_p)_s \frac{\partial \langle T_s \rangle^s}{\partial t} = \lambda_{\text{seff}} \frac{\partial^2 \langle T_s \rangle^s}{\partial x^2} - h_{\text{sf}} a_{\text{sf}} [\langle T_s \rangle^s - \langle T_f \rangle^f]. \quad (2)$$

According to [12], the coefficient of heat exchange between the liquid (gas) and porous body particles can be calculated from the formula

$$\frac{1}{h_{\text{sf}}} = \frac{d}{\text{Nu}_{\text{fs}} \lambda_f} + \frac{d}{\beta \lambda_s}, \quad (3)$$

where $\beta = 10$ if the porous-body particles are spherical. In this expression the value of the Nusselt number for the case of $\text{Re}_p > 100$ correlates well with the expression given in [13]:

$$\text{Nu}_{\text{fs}} = \frac{0.255}{\varepsilon} \text{Pr}^{1/3} \text{Re}_p^{2/3}.$$

The estimates of Nu_{fs} at small values of Re_p vary between 0.1 and 12, as follows from [14, 15].

According to [16], the specific surface of contact between the solid and liquid (gaseous) phases can be calculated as

$$a_{\text{sf}} = \frac{6(1 - \varepsilon)}{d}, \quad (4)$$

and the effective value of the thermal conductivity coefficient for the liquid (gaseous) phase is usually given in the form

$$\lambda_{\text{feff}} = \varepsilon \lambda_f.$$

For a porous body of fine-grain structure the mean particle diameter d is a small quantity, and, according to formulas (3) and (4), the coefficient $h_{\text{sf}} a_{\text{sf}}$ takes large values. This allows one to introduce a small parameter into the formula according to the equation

$$\delta = \frac{1}{h_{\text{sf}} a_{\text{sf}}} \frac{\varepsilon [\langle \rho_f \rangle^f (c_p)_f]^3 \langle v_f \rangle^2}{[\varepsilon \langle \rho_f \rangle^f (c_p)_f + (1 - \varepsilon) \langle \rho_s \rangle^s (c_p)_s] (\lambda_{\text{feff}} + \lambda_{\text{seff}})}.$$

For the method of perturbations to be applied to system (1), (2), we will place it in dimensionless form using the following dimensionless variables:

temperature

$$\Theta = \frac{T - T_0}{T_{\text{in}} - T_0},$$

space

$$\xi = \frac{\langle \rho_f \rangle^f (c_p)_f \langle v_f \rangle}{\lambda_{\text{feff}} + \lambda_{\text{seff}}} x$$

and time

$$\tau = \frac{[\langle \rho_f \rangle^f (c_p)_f \langle v_f \rangle]^2}{[\varepsilon \langle \rho_f \rangle^f (c_p)_f + (1 - \varepsilon) \langle \rho_s \rangle^s (c_p)_s] (\lambda_{\text{feff}} + \lambda_{\text{seff}})}.$$

According to the results of numerical calculations given in [1-3], the difference between the temperatures of the solid and liquid phases is small compared to the difference between the initial temperature of the liquid (gas)

at the entrance to the porous body and the initial temperature of the porous body itself. It is assumed that the solid-phase temperature can be represented as

$$\Theta_s = \Theta_f + \delta \Delta \Theta, \quad (5)$$

and system of equations (1), (2) will be given in the form

$$\frac{\partial \Theta_f}{\partial \tau} + \frac{\partial \Theta_f}{\partial \xi} = \frac{\partial^2 \Theta_f}{\partial \xi^2} + O(\delta), \quad (6)$$

$$\Delta \Theta = \frac{\partial \Theta_f}{\partial \tau} + \Lambda_1 \frac{\partial \Theta_f}{\partial \xi} - \Lambda_2 \frac{\partial^2 \Theta_f}{\partial \xi^2}, \quad (7)$$

where

$$\Lambda_1 = \frac{\varepsilon \langle \rho_f \rangle^f (c_p)_f + (1 - \varepsilon) \langle \rho_s \rangle^s (c_p)_s}{\varepsilon \langle \rho_f \rangle^f (c_p)_f};$$

$$\Lambda_2 = \frac{\lambda_{\text{eff}} [\varepsilon \langle \rho_f \rangle^f (c_p)_f + (1 - \varepsilon) \langle \rho_s \rangle^s (c_p)_s]}{\varepsilon \langle \rho_f \rangle^f (c_p)_f (\lambda_{\text{eff}} + \lambda_{\text{seff}})}.$$

Equation (6) results from the addition of formulas (1) and (2), and Eq. (7) represents relation (1) in dimensionless form with allowance for formula (5).

Solution of the Problem and Analysis. Suppose there is a semi-infinite porous body ($\xi > 0$, where $\xi = 0$ is the boundary) which at the initial instant of time $t = 0$ is at a constant temperature T_0 . Entering through the boundary $\xi = 0$ into the porous body at a constant velocity is an incompressible liquid (gas) whose temperature at the boundary T_{in} is higher than the initial temperature of the porous body. The initial and boundary conditions for the function Θ_f can be presented in the form:

$$\Theta_f(\xi, 0) = 0, \quad \Theta_f(0, \tau) = 1, \quad \frac{\partial \Theta_f}{\partial \xi}(\infty, \tau) = 0. \quad (8)$$

A solution of Eq. (6) subject to initial and boundary conditions (8) is obtained by using the Laplace transform:

$$\Theta_f = \frac{1}{2} \operatorname{erfc} \left\{ \frac{\xi - \tau}{2 \sqrt{\tau}} \right\} + \frac{1}{2} \exp(\xi) \operatorname{erfc} \left\{ \frac{\xi + \tau}{2 \sqrt{\tau}} \right\}. \quad (9)$$

According to Eqs. (7) and (9), the difference between the temperatures of the liquid (gaseous) and solid phases can be determined from the formula

$$\Delta \Theta = \frac{1 - \Lambda_2}{4\tau \sqrt{\pi\tau}} \left\{ (\xi + \tau) \exp \left[- \left(\frac{\xi - \tau}{2 \sqrt{\tau}} \right)^2 \right] + (\xi - \tau) \exp \left[\xi - \left(\frac{\xi + \tau}{2 \sqrt{\tau}} \right)^2 \right] \right\} + (\Lambda_1 - \Lambda_2) \left\{ \frac{1}{2} \exp(\xi) \operatorname{erfc} \left[\frac{\xi + \tau}{2 \sqrt{\tau}} \right] - \frac{1}{2 \sqrt{\pi\tau}} \left[\exp \left\{ - \left(\frac{\xi - \tau}{2 \sqrt{\tau}} \right)^2 \right\} + \exp \left\{ \xi - \left(\frac{\xi + \tau}{2 \sqrt{\tau}} \right)^2 \right\} \right] \right\}. \quad (10)$$

We calculate two physically important integral characteristics of solution (10). It is easily shown that for large values of τ the integral of the temperature difference $\Delta \Theta$ over the semi-infinite porous body is equal to

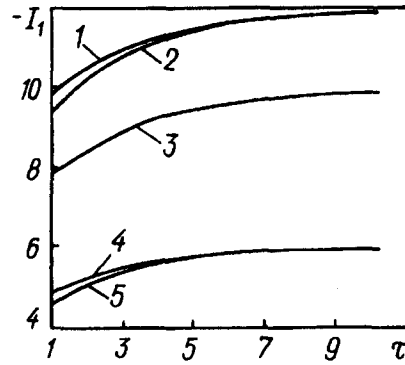


Fig. 1. Dependence of the function $-I_1(\tau)$ on time at different values of Λ_1 and Λ_2 : 1) $\Lambda_1 = 13$, $\Lambda_2 = 4.33$; 2) 13 and 2.6; 3) 11 and 2.2; 4) 7 and 2.33; 5) 7 and 1.4.

$$I_1(\tau) = \int_0^{\infty} \Delta\Theta d\xi = \frac{1 - \Lambda_2}{2\sqrt{\pi\tau}} \exp\left(-\frac{\tau}{4}\right) + \frac{1 - \Lambda_1}{2} \left[1 + \operatorname{erf}\left(\frac{\sqrt{\tau}}{2}\right)\right].$$

The function $-I_1(\tau)$ characterizes the amount of excess energy that cannot be transferred to the porous body by the liquid (gaseous) phase because of the temperature difference between them. We consider this energy a measure of the departure of the process from local thermodynamic equilibrium (when the temperatures of the solid and liquid phases are equal). For a process occurring under local thermodynamic equilibrium, the amount of excess energy is exactly equal to zero. The larger this amount, the larger the departure from equilibrium.

The dependence of the function $-I_1(\tau)$ on time for different values of Λ_1 and Λ_2 (they are determined by the values of the ratios $\langle\rho_f\rangle^f(c_p)_f/\langle\rho_s\rangle^s(c_p)_s$ and $\lambda_{\text{fef}}/\lambda_{\text{seff}}$ and by the porosity ϵ) is shown in Fig. 1. From this figure it follows that the amount of excess energy increases with time until it attains a certain asymptotic value that depends on the thermophysical properties of the solid and liquid phases, porosity, and the mass flow rate of the liquid phase. Returning to dimensional variables, the amount of excess energy per unit area of the boundary surface through which the liquid enters has the form

$$E(\tau) = \int_0^{\infty} (c_p)_f \langle\rho_f\rangle^f [\langle T_f\rangle^f - \langle T_s\rangle^s] dx = - \frac{(c_p)_f \langle\rho_f\rangle^f (T_{\text{in}} - T_0) (\lambda_{\text{fef}} + \lambda_{\text{seff}})}{(c_p)_f \langle\rho_f\rangle^f \langle v_f\rangle} \delta I_1(\cdot).$$

When $\tau \rightarrow \infty$ (correspondingly for $t \rightarrow \infty$), the amount of excess energy tends to

$$\lim_{\tau \rightarrow \infty} E(\tau) = \frac{T_{\text{in}} - T_0}{h_{\text{sf}} a_{\text{sf}}} \frac{(1 - \epsilon) \langle\rho_s\rangle^s (c_p)_s [\langle\rho_f\rangle^f (c_p)_f]^2 \langle v_f\rangle}{[\epsilon \langle\rho_f\rangle^f (c_p)_f + (1 - \epsilon) \langle\rho_s\rangle^s (c_p)_s]}. \quad (11)$$

The second integral characteristic shows the time average of the excess energy amount:

$$I_2 = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^{\infty} \left[\int_0^{\infty} \Delta\Theta d\xi \right] d\tau = 1 - \Lambda_1,$$

and with allowance for dimensional variables

$$\begin{aligned} \bar{E} &= \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^{\infty} \left[\int_0^{\infty} (c_p)_f \langle\rho_f\rangle^f [\langle T_f\rangle^f - \langle T_s\rangle^s] dx \right] dt = \\ &= - \frac{(c_p)_f \langle\rho_f\rangle^f (T_{\text{in}} - T_0) (\lambda_{\text{fef}} + \lambda_{\text{seff}})}{(c_p)_f \langle\rho_f\rangle^f \langle v_f\rangle} \delta I_2 = \end{aligned}$$

$$= \frac{T_{in} - T_0}{h_{sf} a_{sf}} \frac{(1 - \epsilon) \langle \rho_s \rangle^s (c_p)_s [\langle \rho_f \rangle^f (c_p)_f]^2 \langle v_f \rangle}{[\epsilon \langle \rho_f \rangle^f (c_p)_f + (1 - \epsilon) \langle \rho_s \rangle^s (c_p)_s]} \quad (12)$$

Comparison of Eqs. (11) and (12) shows that the mean amount of excess energy in a heated semi-infinite porous body is equal to its amount in the case of $t \rightarrow \infty$.

CONCLUSIONS

1. For the process of heating of a semi-infinite porous body by an incompressible liquid (gas) flow the amount of thermal energy that cannot be transferred by the liquid (gaseous) phase to the porous body because of the temperature difference between the phases is calculated. This energy is called excess energy. It is shown that its amount per unit boundary surface increases with time until it attains a certain constant value which depends on the thermophysical properties of the phases, porosity, and the mass flow rate of the liquid (gaseous) phases.

2. The time average amount of excess energy approaches the same constant value with time.

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NOTATION

a_{sf} , specific surface of contact of the solid and liquid (gaseous) phases, m^2/m^3 ; c_p , specific heat at constant pressure, $J/(kg \cdot K)$; d , mean diameter of the porous-body particles, m ; E , amount of excess energy per unit surface area through which heat is supplied, J/m^2 ; \bar{E} , the same averages over time, J/m^2 ; h_{sf} , coefficient of heat exchange between the porous body particles and liquid (gaseous) phase, $W/(m^2 \cdot K)$; Nu_{fs} , Nusselt number; Re , Reynolds number; Pr , Prandtl number; t , time, sec ; T , temperature, K ; T_{in} , initial temperature of the liquid (gas), K ; v , liquid (gas) velocity, m/sec ; x , coordinate, m ; ϵ , porosity; λ , thermal conductivity, $W/(m \cdot K)$; μ , coefficient of dynamic viscosity, $Pa \cdot sec$; δ , small dimensionless parameter; Θ , dimensionless temperature; ρ , density, kg/m^3 ; τ , dimensionless time; ξ , dimensionless coordinate; $\langle \rangle$, mean-value operation over the volume; $\langle \rangle^f$, phase average over the liquid (gaseous) phase; $\langle \rangle^s$, phase average over the solid phase. Subscripts and superscripts: eff, effective value; feff, effective value for the liquid (gas); in, value at the boundary $x = 0$; f, liquid (gaseous) phase; 0, initial value; s, solid phase; seff, effective value for the solid phase; p, porosity.

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